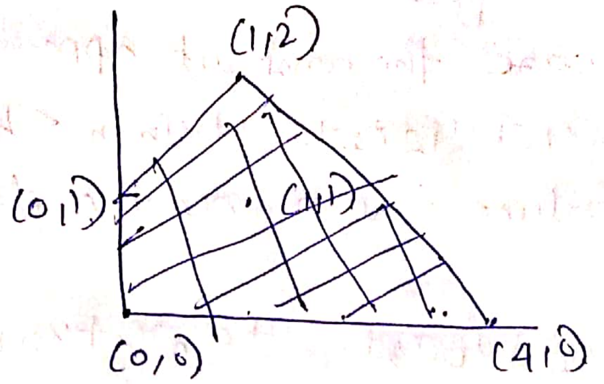
Graph the convex hull of the points
$(0,0)$, $(0,1)$, $(1,2)$, $(1,1)$, $(4,0)$



Shaded region is the convex hull including the boundary.

Standard form of a Linear Programming Problem

Suppose we have two variables and ~~two constraints~~. Then we can write the ~~problem~~ constraints of the problem as

$$a_1x_1 + a_2x_2 \le b_1$$
$$\cancel{a_1'x_1 + a_2'x_2 \le b_2}$$

Then we can add a new variable to make the inequality an equality. Such a variable is known as slack variable. The variable must be positive.

$$a_1x_1 + a_2x_2 + x_3 = b_1$$
$$\cancel{a_1'x_1 + a_2'x_2 + x_4 = b_2}$$

Similarly if the constraint is

$$a_1x_1 + a_2x_2 \ge b_1$$
$$\cancel{a_1'x_1 + a_2'x_2 \ge b_2}$$

then we can subtract a new variable to make the inequality an equality. Here the variable must be positive. Such a variable is known as.

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surplus variable. So we can rewrite the problem as.

$$a_1x_1 + a_2x_2 - x_3 = b_1$$

~~$a_2x_2 + a_3x_3 - x_4 = b_2$~~

For a general case the constraint appears as

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \leq b_i$$

So we can introduce the non negative slack variable as

$$a_{i1}x_1 + a_{i2}x_2 + a_{i3}x_3 + \cdots + a_{in}x_n + x_{n+i} = b_i$$

If the constraint is

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n \geq b_i$$

Then we can introduce the nonnegative surplus variable as.

$$a_{i1}x_1 + a_{i2}x_2 + \cdots + a_{in}x_n - x_{i+n} = b_i$$

So we can standardize an LPP as.

optimize $z' = c_1x_1 + \cdots + c_nx_n$.

Subject to

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + x_{n+1} = b_1$$

$\cdots \qquad \cdots$

$\cdots \qquad \cdots$

~~$a_{i1}x_1 + a_{i2}x_2 + \cdots$~~

$$a_{r1}x_1 + a_{r2}x_2 + \cdots + a_{rn}x_n + x_{n+r} = b_r$$

$$a_{r+1\,1}x_1 + a_{r+1\,2}x_2 + \cdots + a_{r+1\,n}x_n - x_{n+r+1} = b_{r+1}$$

$\cdots \qquad \cdots$

$\cdots \qquad \cdots$

$$a_{s1}x_1 + a_{s2}x_2 + \cdots + a_{sn}x_n - x_{n+s} = b_s$$

$$a_{s+1\,1}x_1 + a_{s+1\,2}x_2 + \cdots + a_{s+1\,n}x_n = b_{s+1}$$

$\cdots \qquad \cdots$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

where the original LPP is

$$\text{optimize } z = c_1x_1 + \cdots + c_nx_n$$

subject to

$$a_{11}x_1 + \cdots + a_{1n}x_n \leq b_1$$

$$\cdots$$

$$a_{r1}x_1 + a_{r2}x_2 + \cdots + a_{rn}x_n \leq b_r$$

$$a_{r+1\,1}x_1 + \cdots + a_{r+1\,n}x_n \geq b_{r+1}$$

$$\vdots$$

$$a_{s1}x_1 + \cdots + a_{sn}x_n \geq b_s$$

$$\cdots$$

$$a_{s+1\,1}x_1 + \cdots + a_{s+1\,n}x_n = b_{s+1}$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

Recorting a LPP.

① All constraints are equations except nonneg
② nonnegativity condition.

② The RHS of each constraint is non-negative

③ All variable nonnegative.

④ Objective function minimization or maximization type.

Ex

$$\text{Max } z = 2x_1 + 3x_2$$

s.to. $x_1 + x_2 \le 5$

$-x_1 + 3x_2 \geq 6$

$x_1, x_2 \geq 0$

Maximize $z = 2x_1 + 3x_2 + 0.x_3 + 0.x_4$

s.to $x_1 + x_2 + x_3 = 5$

$-x_1 + 3x_2 - x_4 = 6$

x_3 = slack

x_4 = surplus

Ex Reduces the following minimization problem, to maximization problem in its standard form

Minimize $z = 3x_1 - 2x_2 + 4x_3$

subject to $x_1 - x_2 + 3x_3 \geq 1$

$2x_1 + 3x_2 - 5x_3 \geq -3$

$4x_1 + 2x_2 \geq 2$

$x_1, x_2, x_3 \geq 0$

So we maximize as

$$z' = -z = -3x_1 + 2x_2 - 4x_3$$

subject to $x_1 - x_2 + 3x_3 \geq 1$

$-2x_1 - 3x_2 + 5x_3 \leq 3$

$4x_1 + 2x_2 \geq 2$, $x_1, x_2 \geq 0$

Introducing slack and surplus variables

$$z' = -z = -3x_1 + 2x_2 - 4x_3$$

subject to $x_1 - x_2 + 3x_3 - x_4 = 1$

$-2x_1 - 3x_2 + 5x_3 + x_5 = 3$

$4x_1 + 2x_2 - x_6 = 2$

$x_1, x_2, x_3, x_4, x_5, x_6 \geq 0$

x_6, x_4 = surplus

x_5 = slack..